A Parametric Approach to Counterparty and Credit Risk

Maximilian Härtel, Giuseppe Orlando
IDS GmbH - Analysis and Reporting Services, Allianz Asset Management AG

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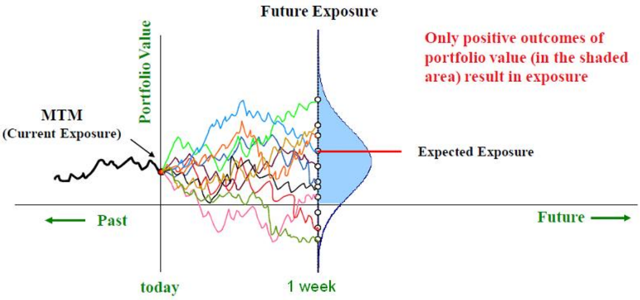
Introduction I

- We want to know the exposure, which is at risk of defaulting counterparties, both today and in the future,
- We want to know our current collateral/margin amounts as well as potential collateral requirements.

Introduction II

- Current exposure is the maximum amount that will be lost if default occurs and if the contract is replaced today.
- Future exposure is the maximum additional amount (under a certain distribution) that will be lost if default occurs on a future date before the maturity of the contract.



Quantification Methods I

The following two methods are used for quantification of future exposure:

1. Regulatory CEM/SM (Current Exposure Method/Standardised Method);
2. Monte-Carlo Simulation (full revaluation).

Quantification Methods II

The Monte-Carlo simulation is the most accurate but it is expensive and cumbersome.

Quantification Methods III

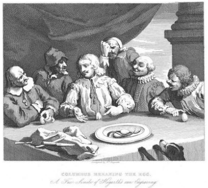
The regulatory approach is easy to implement but has some significant shortcomings:

- It does not differentiate between margined and unmargined transactions;
- It does not sufficiently capture the level of volatilities as observed over the recent stress periods;
- The recognition of hedging and netting benefits through the Net-to-Gross-Ratio is too simplistic and does not reflect economically meaningful relationships between the derivative positions.

N.B.: This method is under review by the regulator and a non-internal model method (NIMM) has been recently proposed in June 2013 (rev. 25 July 2013).

Quantification Methods IV

- Therefore we will explain a third approach, i.e. a parametric methodology.
- Our proposal back in 2011 was to adopt an intermediate model SIMPLE and NOT CUMBERSOME but able to overcome most of the criticism against CEM/SM.



- This is not the egg of Columbus but thanks to its flexibility we could develop on top the concepts of credit loss, default probability (as a result of a Poisson process), we included CCPs (that many believe are risk free) and we extended the framework to liquidity risk by considering potential collateral requirements.

Current Exposure

The Net Replacement Value (NRV) for each counterparty k within each fund and netting group is defined as:

NRV of a counterparty with netting and collateral on a netting group level

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}, 0\right) - \mathrm{CM}_{g,f,k}, 0\right]$$

The current exposure for derivative contracts is defined as the sum of the NRVs across all counterparties:

Current exposure

$$\mathrm{NRV} = \sum_{k=1}^{K} \mathrm{NRV}_k$$

Notation

$\mathrm{NRV}_k :=$ Aggregated Net Replacement Value of counterparty k.
$K :=$ Number of counterparties.
$F_k :=$ Number of funds containing positions with counterparty k.
$G_{f,k} :=$ Number of netting groups in fund f with counterparty k.
$N_{g,f,k} :=$ Number of positions in netting group g, fund f with counterparty k.
$\mathrm{CM}_{g,f,k} :=$ Collateral/Margin amount of counterparty k for fund f and netting group g.
$V_{i,g,f,k} :=$ Value of position i in netting group g, fund f with counterparty k.

Parametric Approach: Assumptions

The maximum credit exposure is the Value at Risk with a 99.5% confidence level.

Arbitrary assumptions for annual volatility, delta and duration for VaR calculation

Assumptions for the extended VaR Matrix										
	Equity		Commodity		FI / IR		Credit		FX	
	Fut/Fwd	Opt	Fut/Fwd	Opt	Fut/Fwd Swp	Opt Swpt	Fut/Fwd Swp	Opt Swpt	Fut/Fwd	Opt
Vola	30%	30%	30%	30%	5%	5%	40%	40%	10%	10%
Delta	1.00	0.50	1.00	0.50	1.00	0.50	1.00	0.50	1.00	0.50

For interest rate/fixed income derivatives, we assume the following VaR factors (increasing with maturity)

Assumptions for the extended VaR Matrix			
Time to Maturity	$<$ 1 year	1 - 5 years	$>$ 5 years
Time Factor	1	3.5	10

Parametric Approach: Comments

- For sake of simplicity, we assume that the distribution is normal (a more refined approach, e.g. the Cornish-Fisher expansion, can be used in case the skewness and kurtosis are calculated). Hence $\alpha = 2.3226$ (one-sided 99.5% confidence level).
- In this context, the VaR is the Shortfall/CVaR for a lower confidence level.

Parametric Approach: Calculations

For collateralised positions we calculate the weekly VaR.

VaR factor for collateralised positions

$$\text{VaR}_{weekly} = \alpha \cdot \frac{\text{Vol}_{annual}}{\sqrt{52}} \cdot \text{Delta} \cdot T$$

For uncollateralised positions we calculate the fortnightly VaR.

VaR factor for uncollateralised positions

$$\text{VaR}_{fortnightly} = \alpha \cdot \frac{\text{Vol}_{annual}}{\sqrt{26}} \cdot \text{Delta} \cdot T$$

For example, the VaR factor for an uncollateralised interest rate swap with time to maturity in 4 years is 8%.

VaR factor for an interest rate swap with maturity in 4 years

$$\text{VaR}_{fortnightly} = 2.3263 \cdot \frac{0.05}{\sqrt{26}} \cdot 1.00 \cdot 3.5 = 0.08$$

Potential Future Exposure without Diversification I

The potential NRV of a counterparty k is the future exposure with counterparty k denoted by NRV-VaR$_k$, where VaR represents the amount calculated as follows:

VaR amount depending on the notional

$$\text{VaR}_{i,g,f,k} = \text{Not}_{i,g,f,k} \cdot \text{VaRFactor}_{i,g}$$

VaRFactor$_{i,g}$:= VaR factor of position i in netting group g.
VaR$_{i,g,f,k}$:= VaR of position i in netting group g, fund f and with counterparty k.
Not$_{i,g,f,k}$:= Notional of position i in netting group g, fund f and with counterparty k.
NRV-VaR$_k$:= Aggregated Net Replacement Value including VaR of counterparty k.

Potential Future Exposure without Diversification II

The VaR factor depends on the investment type. This amount is used for computing the potential future exposure as shown in the formula below:

VaR amount depending on the notional

$$\text{NRV-VaR}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k} + \text{VaR}_{i,g,f,k}, 0\right) - \text{CM}_{g,f,k}, 0\right]$$

$\text{VaR}_{i,g,f,k} :=$ VaR of position i in netting group g, fund f and with counterparty k.
$\text{CM}_{g,f,k} :=$ Collateral/Margin amount of counterparty k for fund f and netting group g.
$\text{NRV-VaR}_k :=$ Aggregated Net Replacement Value including VaR of counterparty k.

Probability of Default

- We define the Credit Loss (CL) as the expected loss for the investor;
- There are two distinct credit losses: the CL based on the current exposure and that on the future exposure with the respective counterparty;
- In both cases, we need the probability of default of the respective counterparty which we derive from its rating;
- Historic data is only available on a yearly basis;
- The credit event is the first event of a Poisson counting process. With this assumption we can calculate the probability of defaults for time horizons of one or two weeks and denote it by p_k for the respective counterparty k.

Credit Loss I

CL of a position based on current exposure

$$\mathrm{CL}_{i,g,f,k}^{curr} = V_{i,g,f,k} \cdot \mathrm{LGD} \cdot p_k$$

CL of a position based on future exposure

$$\mathrm{CL}_{i,g,f,k}^{future} = (V_{i,g,f,k} + \mathrm{VaR}_{i,g,f,k}) \cdot \mathrm{LGD}_i \cdot p_k$$

Credit Loss II

CL with a counterparty based on current exposure

$$CL_k^{curr} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} CL_{i,g,f,k}^{curr}, 0\right) - CM_{g,f,k}, 0\right]$$

CL with a counterparty based on future exposure

$$CL_k^{future} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} CL_{i,g,f,k}^{future}, 0\right) - CM_{g,f,k}, 0\right]$$

CL

$$CL^{curr} = \sum_{k=1}^{K} CL_k^{curr} \qquad CL^{future} = \sum_{k=1}^{K} CL_k^{future}$$

Unexpected Loss I

Definition (Unexpected loss (UL))

We define the Unexpected Loss (UL) as the variation in the expected loss. UL is calculated as a standard deviation from the mean (given a certain confidence level) or, equivalently, as the difference between the expected loss (EL) and the VaR at a given confidence level. It is used to determine the level of economic capital to be held.

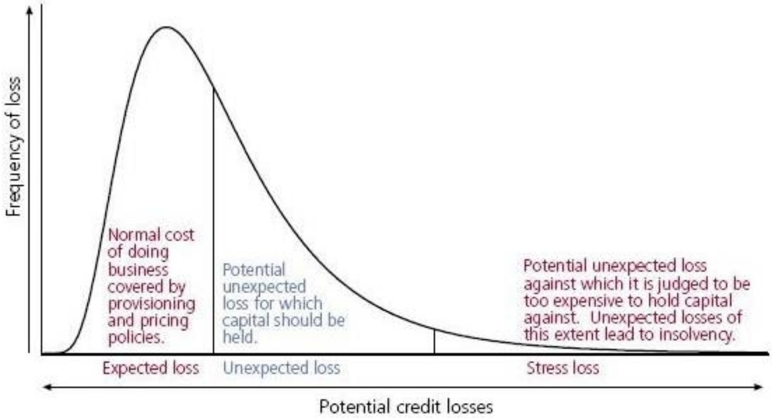
UL of a position based on current exposure

$$\mathrm{UL}_{i,g,f,k}^{curr} = \sqrt{V_{i,g,f,k}^2 \cdot \mathrm{LGD}^2 \cdot p_k \cdot (1 - p_k)}$$

UL of a position based on future exposure

$$\mathrm{UL}_{i,g,f,k}^{future} = \sqrt{(V_{i,g,f,k} + \mathrm{VaR}_{i,g,f,k})^2 \cdot \mathrm{LGD}^2 \cdot p_k \cdot (1 - p_k)}$$

Unexpected Loss II



Unexpected Loss III

UL with a counterparty based on current exposure

$$\mathrm{UL}_k^{curr} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{UL}_{i,g,f,k}^{curr}, 0\right) - \mathrm{CM}_{g,f,k}, 0\right]$$

UL with a counterparty based on future exposure

$$\mathrm{UL}_k^{future} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{UL}_{i,g,f,k}^{future}, 0\right) - \mathrm{CM}_{g,f,k}, 0\right]$$

UL

$$\mathrm{UL}^{curr} = \sum_{k=1}^{K} \mathrm{UL}_k^{curr} \qquad \mathrm{UL}^{future} = \sum_{k=1}^{K} \mathrm{UL}_k^{future}$$

Correlation I

- If there is a change in the value of a derivative this might impact the value of other derivatives of this counterparty.
- Correlation can be included only for positions within a netting group because positions from different netting groups cannot be netted:

$$\Sigma_{g,f,k} := \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{pmatrix}$$

- c_{kl} denotes the correlation between position k and position l of netting group g.
- The weight $w_{i,g,f,k}$ of each position i in a netting group must be considered and it is measured by the respective notional.

Correlation II

Then, we define the vector $v_{g,f,k}$ as $w_{g,f,k}^{\top} \cdot \beta_{g,f,k}$ with:

Weight vector of netting group g

$$w_{g,f,k} := \left(w_{1,g,f,k}, \cdots, w_{n,g,f,k}\right)^{\top}$$

VaR-factor vector of netting group g

$$\beta_{g,f,k} := \left(\text{VaRFactor}_{1,g,f,k}, \cdots, \text{VaRFactor}_{n,g,f,k}\right)^{\top}$$

We obtain the VaR factor for the netting group g as :

VaR-factor

$$\text{VaRFactor}_{g,f,k} := \sqrt{v_{g,f,k}\ \Sigma_{g,f,k}\ v_{g,f,k}}$$

Correlation III

In the end, the future exposure of a counterparty k, i.e. the aggregated NRV including the VaR amount of counterparty k with integration of correlation, is:

Net Replacement Value with correlation

$$\text{NRV-VaR}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}, 0\right) + \text{VaR}_{g,f,k} - \text{CM}_{g,f,k}, 0\right]$$

Potential Collateral Requirements I

When the value of a position decreases, it not only has an impact on the NAV but on the collateral too. Three cases might occur:

1. The position changes from positive to negative i.e. the fund has to give back the collateral received and post collateral;
2. The position remains positive, i.e. the fund has to give back part of the collateral received;
3. The position remains negative, i.e. the fund has to post additional collateral.

Potential Collateral Requirements II

We define the Potential Collateral Requirements (PCR) of counterparty k as:

PCR with a counterparty k

$$\mathrm{PCR}_k = \sum_{f=1}^{F_k} \max\left[-\sum_{g=1}^{G_{f,k}} \min\left(V_{g,f,k} + \mathrm{VaR}_{g,f,k} - \mathrm{CM}_{g,f,k}, 0\right), 0 \right]$$

Potential Collateral Requirements III

- The difference between available assets (i.e. liquidity, borrowing and eligible assets for collateral) and PCRs can be used for managing liquidity risk as a shortfall measure.
- The collateral amount of a certain netting group collateral can be positive or negative, i.e. broker and client collateral have to be taken into consideration, respectively.
- In order to evaluate, whether the eligible assets of a fund are sufficient to cover potential collateral requirements, the PCR must be reduced by the amount of off-balance broker collateral before comparing it to the amount of assets available.